A Method of Multi-objective Decision Functions Based on GP Algorithm Theory

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Abstract. In multi-objective decision problems many values must be assigned, such as the importance of the different objective and the values of the alternatives with respect to subjective objective. In this paper we propose a new construct method of multi-objective decision functions based on genetic programming algorithm, which produces more stable decision functions than the AHP arithmetic mean used ones. The theoretical expectations are validated by case studies.

1. Introduction

There are four objectives to solve multi-objective decision-making problems: (1) deleting the poor feasible program; (2) choosing many best options; (3) arranging the feasible programs; (4) choosing and arranging the integrated goals. The programs of deletion and choice are the most important in these programs.

The purpose of this paper is to construct a decision-making function of the fixed problem, and search a possible stable solution to the whole decision functions. The decision-making function is a function satisfied with some nature axioms. The general formalization of these functions can be denoted with the style of tree, and the leaves are the power-style decision function with the weight. The preceding of the leaves is the average value of power. The value of the function is at the root of the tree, and the value is calculated by recursion. The value of non-leaf is the average power of its sub-structure. The paper will propose a possible stable decision-making function for some multi-objective decision problem by the genetic programming algorithm[1][14][18]. The unit of the genetic programming algorithm is decision-making function denoted by structure tree. Formally, the sub-tree is a decision-making function. If the problem is a group-decision problem, we may think the group-decision function is a clustering of many individual decision-making functions. In this paper, we only discuss about the problem of one decision-maker and many rules, and don't discuss about the clustering of many decision-making function.

Through analyzing the sensitivity of multi-objective decision-making function, we can analyze the result of the decision. Presently, there are many measures on the analysis of the sensitivity referred by literatures when there is the minor perturbation on the value of the feasible program and weight. Typically, there are the PROMETHEE method[2][3][15] for more than one parameter and the WINGDSS method for all parameters[3]. But these methods only adapt to analyze the sensitivity of simple decision-making function optimized by linear programming[4][6][7][11], and these methods are not adaptive to the complex decision-making functions which is tree-style and from the multiple objective decision-making program. An optimized method based on the genetic program is proposed in the paper.

In this paper, we will introduce into an overall stable index in the scope of (0, 1), and use it to measure the stability of the solution. If the index is 1, then the solution is stable. When the value of the index is closer to 1, the solution is more stable. The overall stable index is the local stable function. The overall stable index is 1 only when all the local stable indexes are 1. The function may be the average with the power , or the other average. The literature [5][8][9][13] lists the local stable indexes, and in our paper we put forward that the solution should be penalized when its local

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stable index is 0 or close to 0. If one stable index is 0, then the value of the overall stability is 0. There are two means of the average power to satisfy the condition. They are the mean (minimal function) of power when , and the mean (geometric mean) of power when . In this paper, we hope to punish the solution of the average stable index which is close to 0. So we choose the geometric mean.

2. Multi-objective decision-making problem

Define 1: In a multi-objective decision-making problem, there are n kinds of feasible programs, $A_1,\ldots,A_n$, which is arranged by the weights $\omega_1,\ldots\omega_n$ with the rules $C_1,\ldots,C_m$. The decision-making Table is composed of utility values which are derived from every feasible program with each rule as follows:

$$\begin{array}{cc} & \begin{array}{ccc} x_1 & \cdot\cdot & x_n \\ A_1 & \cdot\cdot & A_n \end{array} \\ \begin{array}{cc} w_1 & C_1 \\ w_2 & C_2 \\ w_m & C_m \end{array} & \begin{bmatrix} a_{11} & \cdot\cdot & a_{1n} \\ \vdots & \cdot\cdot & \vdots \\ a_{m1} & \cdot\cdot & a_{mn} \end{bmatrix} \end{array} \tag{1}$$

In the Table, aij＞0 denotes the utility value of the i-th rule and the j-th feasible program. xj denotes the integrative utility value of all the j-th rules. { x1,…,xn} denotes the solution of the multi-objective problem. The feasible programs rules by the value of xj. i.e. the max is xj, the best is Aj.

If the utility value of the feasible program with different objective is not developed with a range of scales, the different objective may cause the different solution. How to develop this scale is not clear. There are many multi-objective decision-making methods using scaling, and the characteristics of a rule could give an appropriate tips of scale. In this paper, the scale is based on the utility function similar to the function of Promethee[9[[10]117]. We maybe use other scale, and use the scale only to simple the problem. If the value of xj is larger, then the i-th rule is instead by

$$a_{ij} := 8\frac{a_{ij} - \min\{a_{ik} : k = 1,\cdots,n\}}{\max\{a_{ik} : k = 1,\cdots,n\} - \min\{a_{ik} : k = 1,\cdots,n\}} + 1 \tag{2}$$

If the value of xj is smaller, then i-th rule is instead by

$$a_{ij} := 8\frac{\max\{a_{ik} : k = 1,\cdots,n\} - a_{ij}}{\max\{a_{ik} : k = 1,\cdots,n\} - \min\{a_{ik} : k = 1,\cdots,n\}} + 1 \tag{3}$$

The utility value of the feasible program of different rule is designated in the scale [1-9] range by this way. If the utility value is larger, then the minimal value of the rule Ci is 1 and the max value of the rule Ci is 9; or else the minimal value of rule Ci is 9 and the max value of the rule Ci is 1. Compared the utility value of the different rules, the scale range is important. At the same time, the feasible program Ai, which is based on the rule Ci, is better when the aij is larger. The integrated utility value of xi is the function of the weight wi and the value aij, i∈{1,…,m}. The same function must adapt to all the feasible programs, and the function is called by the decision-making function.

To enable the decision-making function has a practical solution, this function should satisfy some natural conditions, which is the axiom of the decision-making function (as follow the define 2).

Define 2: If the axiom is true, the function, $f : R_+^m \times R_+^m \to R_+$, is a decision-making function.

Axiom A1: f= f(w,a)satisfy the condition

$$f(\lambda w, a) = f(w,a) \text{ and } f(w,\lambda a) = \lambda f(w,a),\ w.a \in R_+^m,\ \lambda \in R_+ . \tag{4}$$

Axiom A2: $f(w\sigma(1),\cdots,w\sigma(m); a\sigma(1),\cdots,a\sigma(m)) = f(w_1,\cdots,w_m; a_1,\cdots,a_m)$

$$w_1\cdots,w_m,a_1,\cdots,a_m \in R_+,\ \sigma \in \{1,\cdots,m\} \tag{5}$$

Axiom A3: Function f is monotonically increasing with every variable ai (i=1,…,m).

Axiom A4: $\min_{i=1,\cdots,m} a_i \le f(w_1,\cdots,w_m; a_1,\cdots,a_m) \le \max_{i=1,\cdots,m} a_i$. (6)

x_j is calculated by the formula $x_j = f(w_1, \cdots, w_m; a_{1j}, \cdots, a_{mj})$, (f is a decision-making function).

The nature type of the decision-making function is the average of power α, that is

$\alpha \geq 0$ (7)

For example, the arithmetic mean is adopted in the AHP, ($\alpha = 1$). When a subjective value is applied in multi-objective decision-making, the choice of a decision-making function of a stable solution is particularly important.

Define 3: When there is a small perturbation to the weight w_i or the value a_{ij} in the decision-making Table, the solution $\{x_1, \ldots, x_n\}$ is stable on the perturbation if the new descending ordering solution $\{x_1', ..., x_n'\}$ influenced by the perturbation is consistent to the descending ordering solution $\{x_1, \ldots, x_n\}$.That means there is perturbation σ as $x\sigma_1 \geq \cdots \geq x\sigma_n$ and $\sigma x_1' \geq \cdots \geq \sigma x_n'$.

3. Sensitivity with respect to alternatives

In the sensitivity with respect to the feasible programs, when i,j {1,2,…n}, the utility value of the i-th rule and the j-th feasible program will be in the range of , , and aij is the estimated value of the i-th rule and the j-th feasible program when the initial value is allowed to change etimes. The integrative utility value of the i-th feasible program will be in the scope of by the interval arithmetic expression. The precise mathematical denotation is described as the definition 4. Simply, we only discuss about the complexity situation when there is no function relationship among ε, i and j in this paper. The choosing ofε is a key in the following method.

Define 4: If $\varepsilon > 0$, then in the feasible program A_j,

$$x_j^-(\varepsilon) = \min\left\{ f(w, \bar{a}) : \bar{a} \in \prod_{i=1,\cdots,m} [a_{ij} - \varepsilon a_{ij}, a_{ij} + \varepsilon a_{ij}] \right\} \quad (8)$$

is the smallest integrated utility values under the condition of perturbation, and

$$x_j^+(\varepsilon) = \max\left\{ f(w, \bar{a}) : \bar{a} \in \prod_{i=1,\cdots,m} [a_{ij} - \varepsilon a_{ij}, a_{ij} + \varepsilon a_{ij}] \right\} \quad (9)$$

is the largest integrated utility values under the condition of perturbation. The sensitivity analysis of the utility of the feasible program is the calculation of these final values.

Proposition 1: The smallest value, $x_j^-(\varepsilon)$, (or the largest value, $x_j^+(\varepsilon)$,) of a decision-making function is the solution of $\bar{a} = (a_{ij} - \varepsilon a_{ij})_{i=1,\cdots,m}$,(or $\bar{a} = (a_{ij} + \varepsilon a_{ij})_{i=1,\cdots,m}$).

Demonstration: the strict monotonicity of the decision-making function could be used to prove the conclusion directly. According to the definition of the stability of the value previously, we can also calculate the stable value the feasible program in descending order.

Define 5: Assuming the integrated value of the feasible program, x_j, by descending order, giving the perturbation σ similar to $x\sigma(_1) \geq \cdots \geq x\sigma(_n)$, the stability of the list $x\sigma_{(j)} \geq x\sigma_{(j+1)}$ is the largest $\delta \in [0,1]$, $x\sigma_{(j)}^-(\delta\varepsilon) \geq x\sigma_{(j+1)}^+(\delta\varepsilon)$, i.e. if there is the largest perturbation σ in the feasible program, the order of the feasible programs $A\sigma_{(j)}$ and $A\sigma_{(j+1)}$ remains unchanged. So the feasible program is the most stable under the largest perturbation ε when the stable value is 1. We call the ε as the stable indicator decision-making function f (a fixed decision-making Table), $S(\varepsilon) = (\prod_{j=1}^{n-1} \delta_j)^{\frac{1}{n-1}}$. The solution is stable under the largest perturbation ε when the stable indicator is 1. The solution is instable when the stable indicator is 0, and the solution becomes more stable when the indicator is more close to 1.

4.GP algorithm theory

How to find the most stable decision-making function of any given multi-objective is, in fact, the regression problem of function. I.e. how to find the stable sort of the feasible programs which could meet the axiom of the decision-making function, which is introduced previously, and a given multi-objective decision-making problem. Genetic programming is suit to resolve the precise structure of unknown function. The reason is: (1) genetic programming can produce a full range of space in the program as an evolutionary approach; (2) their individual structure evolves with their connotations; (3) If the appropriate denotation is selected, the solution, which is better than the artificial solution, is found easily. Genetic programming algorithm applies the user-defined functions and reuses the dynamic layering process, which is the expansion of genetic algorithm. As one of the most important features of a GP, the individual of the groups generally use the dynamic tree structure. The node of the tree is composed with terminal set, primitive functions set and operator. GP-searching space is the tree space including a choice function set and the terminal set.

Algorithm 1: GP algorithm

Define:

f : evaluating function, that is used to obtain the value of an individual's fitness by evaluate the unit (computer program), $p \in N_{pop}$.

the threshold value of an individual fitness, which is the qualification for judging whether or not the end of the evolution.

size, i.e. the number of the units in a group

γ : the proportion of individuals of generation crossover operator in each groups in the evolutionary process (crossover probability).

μ : mutation probability

λ : the termination conditions of the separate evolution, λ denotation that the termination condition has not yet reached.

The general algorithm of GP may be described as follow: GP(f,,,γ,μ,λ);

Step 1: Initialize groups: ←, randomly generate computer programs (spanning tree);

Step 2: Compute f(p) for all $p \in N_{pop}$;

Step 3: if((max{ f(p)| $p \in N_{pop}$ })<)$\wedge$(<λ)) { generate a new group,

(1) Selecting (1-γ) individualities from N_{pop}, and inject with ` . The probability that the individuality $p \in N_{pop}$ is selected is $(p) = \frac{f(p)}{\sum_{i=1} f(p_i)}$, or calculated by the way of league selection.

(2) According to the probability values from the choice (1), selecting individual γ from the group N_{pop} , and using the crossover operator each other, then putting the next generation in ‘

(3) Select μ individuals from the group ` , and mutate them.

(4) Replace (← ``)with `.

(5) calculate f(p) for all $p \in N_{pop}$.

Step 4 select the largest unity $p \in N_{pop}$ from f(p) as the best unity.

When we choose the mechanism of notation, the function, which has a feasible solution, must be considered in the space of functions, and the function satisfies the axiom of the decision-making function. We select the coding method only on the feasible solution in the paper. At first, construct an effective decision-making function from the initial decision-making functions, i.e. the mean of the power α ,($\alpha \geq 0, \alpha = 0$, getting the limited geometric mean).

According to Proposition 2, it is easy to find tree-based GP systems are all the individual decision-making function. On the other hand, we can not cover all possible decision-making function by use of this expression, but we still have access to more general decision-making function than the initial decision-making function. The fitness evaluation is strictly arising from the sensitivity analysis of the multi-objective decision-making problems.

Algorithm 2: The fitness algorithm of evolution and decision-making function

(1)Calculating $x_j, x_j^-(\varepsilon), x_j^+(\varepsilon)$ for each feasibility program A_j ;

(2)Descending order for each value of the combined effect of the feasible program;

(3)Calculating the stable value δ_j of each pair of adjacent $A\sigma_{(j)}$ and $A\sigma_{(j+1)}$;

(4)Calculation of fitness as a stability indicator, and fitness= $S(\varepsilon)$.

5. Sensitivity with respect to weights

We assume that the weight of different objective is a constant previously. The utility value of the feasible program is assumed to be constant (in the interval of [1,9]) on the objective.

Define 6: For each feasible program A_j,

$$x_j^-(\varepsilon) = \min\left\{ f(\bar{w}, a) : \bar{w} \in \prod_{i=1,\cdots,m} [w_i - \varepsilon w_i, w_i + \varepsilon w_i] \right\} \quad (10)$$

$$x_j^+(\varepsilon) = \max\left\{ f(\bar{w}, a) : \bar{w} \in \prod_{i=1,\cdots,m} [w_i - \varepsilon w_i, w_i + \varepsilon w_i] \right\} \quad (11)$$

are the minimum and maximum integrated utility value respectively under the conditions of perturbation permitted. The sensitivity analysis on weight is composed of $[x^-(\varepsilon), x^+(\varepsilon)]$ found. Here $x^-(\varepsilon) = (x_1^-(\varepsilon), \cdots, x_n^-(\varepsilon))$ and $x^+(\varepsilon) = (x_1^+(\varepsilon), \cdots, x_n^+(\varepsilon))$.

Define 7: Assuming the integrated utility values of the feasible programs is in descending order, given the perturbation σ similar to $x\sigma_{(1)} \geq \cdots \geq x\sigma_{(n)}$. At the same time assuming $w = (w_1, \cdots, w_m)$, $v = (v_1, \cdots,, v_m)$, $a_j = (a_{1j}, \cdots, a_{mj})$, The stability of the sequence $x\sigma_{(j)} \geq x\sigma_{(j+1)}$ is the largest $\delta \in [0,1]$, and $f(v, a\sigma_{(j)}) - f(v, a\sigma_{(j+1)}) \geq 0$. Here $v \in [w - \delta\varepsilon w, w + \delta\varepsilon w]$. I.e. if we permit the weight of the largest perturbation, the order of the feasible program $A\sigma_{(j)}$ and $A\sigma_{(j+1)}$ remains unchanged. Therefore, the value of stability, 1, represents the sort is the most stable under the maximum perturbation. We call ε as an indicator of stability of the decision-making function f (fixed decision-making Table), and $S(\varepsilon) = (\prod_{j=1}^{n=1} \delta_j)^{\frac{1}{n-1}}$.

It should be emphasized the role of weight because there is a nonlinear optimization problem, rather than the feasible program has a direct linear optimization in a range. In the GP system, the nonlinear optimization algorithm is very slow and inefficient. The basic idea of algorithm is to reduce the global optimization problem by the technique of branch and bound, and solve the optimization problem of monotone. Because the decision-making function is "monotone clustering" of the average power of the weight, it relates to the field of simplex optimization problem.

6. Experimental results and analysis

Table 1 The set of parameters in GP system.

Target	the greatest stability of the evolution of decision-making function
Endpoint set	$\forall \alpha \in [0,40]$,the average of the power α being evaluated
Function set	the average of the power α is equal to the weigth of α -th power, $\alpha \in [0.40]$
Adaptation	The feasible program of the multi-objective decision-making problem
fitness	Calculate the indicators of stability given a multi-objective decision-making problem
Species size	500,1000,2000
Crossover probability	90%
Mutation probability	10%
Selection Method	Tournament selection, the number is 5
The standard of termination	nothing
The largest number of generations	40,50
The maximum depth of cross-tree	10
Initial method	Reproduction

In the following experiment, we consider to purchase the best TV which has been given prices by 10 different suppliers. The subjective decision-making objective is in ascending order by the importance, we analyze the stability of the utility value of the feasible program. That is a multi-objective decision-making problem with 4 rules and 10 kinds of feasible programs:

Assuming the changes in the utility value of the feasible program can reach 10%. In GP system, the set of experimental parameters, which includes control parameters, is shown in Table 1.

Decision-making Table and the value of set subjective objective is list in the following:

$$\begin{array}{ll} & \begin{array}{cccccccccc} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 & A_9 & A_{10} \end{array} \\ \begin{array}{ll} w_1 = 0.1 & c_1 = Aesthetic-guideline \\ w_2 = 0.2 & c_2 = Warranty-guideline \\ w_3 = 0.3 & c_3 = Brand-guideline \\ w_4 = 0.4 & c_4 = \mathrm{Im}\,age-guideline \end{array} & \begin{bmatrix} 1 & 2 & 9 & 3 & 1 & 4 & 8 & 9 & 5 & 6 \\ 3 & 9 & 1 & 6 & 9 & 7 & 8 & 2 & 3 & 1 \\ 9 & 1 & 7 & 1 & 8 & 8 & 3 & 6 & 2 & 9 \\ 5 & 6 & 2 & 9 & 1 & 2 & 4 & 3 & 7 & 4 \end{bmatrix} \end{array} \quad (12)$$

In Table 2, each line shows the characteristics of each decision-making function generated by the stability, the tree nodes, the number of generations and the species size according to the GP algorithm. The results showed that the more stable solution may be obtained from the larger species. The result is better when the number of species is 2000, rather than the species is 4000. The best indicators of stability of the decision-making function is . This shows that a stable solution may be obtained when the average of perturbation drops to 21.6 % perturbation permitted initially. The best structure of the decision-making function is found in Figure 1. We evaluate each internal node by using the corresponding average of power   of progeny (equal weights), and evaluate each leaf node by using the corresponding utility value of the feasible program. Then the decision-making function can be described as follow:

$$f = \left(\frac{\left(\left(\frac{m_{0.6}^{2} + m_{0.1}^{2}}{2} \right)^{\frac{1}{2}} \right)^{10} + m_{0.1}^{10}}{2} \right)^{\frac{1}{10}} \quad (13)$$

The solution reached is not very stable through this decision-making function (when the stability is $S(\varepsilon)=1$, the solution is most stable). There are two solutions in Table 3, one solution is in descending order.

Table 2 The result of GP program.

#	stability	size	generation	species size
1	0.163	3	7	500
2	0.164	7	12	500
3	0.167	11	9	500
4	0.168	15	22	500
5	0.167	23	17	1000
6	0.167	19	8	1000
7	0.169	19	17	1000
8	0.166	19	19	2000
9	0.169	11	19	2000
10	0.216	5	12	2000

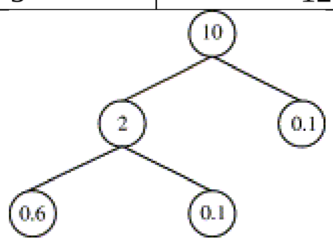

Figure 1 The structure of decision-making function.

Table 3 The solution of GP compared with the solution of arithmetic mean.

The solution of GP										
Order	A_1	A_7	A_6	A_{10}	A_4	A_9	A_8	A_2	A_5	A_3
x_j	4.796	4.602	4.377	4.312	4.277	4.060	3.914	3.754	3.385	3.204
δj	0.213	0.252	0.092	0.060	0.287	0.197	0.240	0.678	0.357	
Arithmetic mean										
Order	A_1	A_4	A_{10}	A_6	A_7	A_2	A_5	A_9	A_8	A_3
x_j	5.4	5.4	5.1	5.0	4.9	4.7	4.7	4.5	4.3	4.0
δj	0	0.389	0.1	0.1	0.256	0	0.225	0.252	0.404	

For each solution, the utility-combined value of the feasible program, xj, at the same time lists the value of stability, according to each pair of consecutive feasible program. In the list of the arithmetic mean, there are two pairs of the entire unstable solution in the sequence of the feasible program, i.e. A_1,A_4, and A_2, A_5. Therefore, the conclusion from the evolution of decision-making function tells us purchasing TV using A_1 programs. The conclusion isn't based on aesthetic objective, but more than brand image objective. We can not distinguish between program A_1 and program A_4 through the arithmetic mean the program.

7. Conclusions and discussions

This paper presents a novel method to construct multi-objective decision-making function based on genetic program algorithm (GPA). The decision-making function, which is constructed by using the method, has obvious better stability than the decision-making function constructed by the typical AHP (arithmetic mean) method. The method proposed in this paper also is the expansion of AHP method. The experimental results show that: the decision-making function by using the genetic programming algorithms is more stable than the decision-making function by using the arithmetic mean decision-making function.

In this paper, we discuss the decision-making function which only has a single decision-maker. The research on the clustering decision-making function of the group decision-making will be a very interesting. The new research will be further interesting to realize the structured decision-making program of the complex multi-objective group decision-making.

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